Abraham-Solution to Schwarzschild Metric Implies That CERN Miniblack Holes Pose a Planetary Risk

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Abstract

A recent mathematical re-interpretation of the Schwarzschild solution to the Einstein equations implies global constancy of the speed of light c in fulfilment of a 1912 proposal of Max Abraham. As a consequence, the horizon lies at the end of an infinitely long funnel in spacetime. Hence black holes lack evaporation and charge. Both features affect the behavior of miniblack holes as are expected to be produced soon at the Large Hadron Collider at CERN. The implied nonlinearity enables the "quasar-scaling conjecture." The latter implies that an earthbound minihole turns into a planet-eating attractor much earlier than previously calculated – not after millions of years of *linear* growth but after months of *nonlinear* growth. A way to turn the almost disaster into a planetary bonus is suggested.

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"Mont blanc and black holes: a winter fairy-tale." What is being referred to is the greatest and most expensive and potentially most important experiment of history: to create miniblack holes [1]. The hoped-for miniholes are only 10^{-32} cm large – as small compared to an atom (10^{-8} cm) as the latter is to the whole solar system (10^{16} cm) – and are expected to "evaporate" within less than a picosecond while leaving behind a much hoped-for "signature" of secondary particles [1]. The first (and then one per second and a million per year) miniblack hole is expected to be produced in 2008 at the Large Hadron Collider of CERN near Geneva on the lake not far from the white mountain. The smashing of ultra-fast protons against each other at sufficient beam accuracy is expected to do the trick with the help of string theory [1].

The experiment is controversial for quite a while. After concern was raised on the Internet that the miniholes might "eat the earth" [2], a worst-case scenario was calculated for the case that they for some reason do not evaporate [3,4] since the expected Hawking evaporation [5] is not a measured fact and Helfer [6] has put it in doubt in a learned paper. Fortunately, the small mass of the miniholes (ideally a multiple of the Planck mass of 0.05 milligram – a grain of salt – but actually 15 orders of magnitude less [4]) endows them with a very small attractive cross section indeed. So they can only gobble up about one point-shaped quark (with appendages) per day [4]. This argument implies that humanity in the case of non-evaporation still has much more than a million years left to evacuate the planet before the latter is "eaten" [4] – an outlook not too much different from the four billion years still allotted to us by the sun.

Thus, the currently accepted "safety net" can be described as having 4 levels:
(i) the miniholes may fail to appear;
(ii) the overwhelming majority will leave the earth immediately;
(iii) all will evaporate;

(iv) if not, a long period of *linear* growth – "1 quark per week eaten" – lets them be pussycats since it will take at least a million years before they "eat the earth" [3,4].

This information has effectively calmed down the Internet community.

Is this "appeasement philosophy" (as Einstein might have said) justified? First, Helfer's nonevaporation conjecture [6] could recently be proved independently [7]. Second, a new conjecture (the quasar scaling conjecture) becomes possible as a consequence leading to the conclusion of the title as we shall see.

Helfer's conjecture is confirmed by a new result in general relativity. General relativity could be changed for once – not in its mathematics but in the interpretation of the mathematics [7]. Two new implications follow: lack of evaporation and lack of chargedness of black holes. The second feature then via nonlinear self-organization leads to a new prediction ("fast earth-eating").

Specifically, the Einstein equations imply the famous Schwarzschild solution which describes how light and everything else behaves in greater and greater proximity to a heavy mass if the latter is condensed down to its Schwarzschild radius. Into the radial Schwarzschild metric one can, for example, enter the simultaneous positions of two stationary outside points r_o and r_i ("*o*uter" and "*i*nner") while r is the radial parameter (not the radial distance) and $r_s = 2GM/c^2$ is the Schwarzschild radius (with G being Newton's gravitational constant, M the gravity-generating mass in question and c the standard speed of light). The metric then allows one to calculate the so-called "coordinate time difference" Δt for light sent between points r_i and r_o and vice versa,

$$\Delta t = \frac{1}{c}\int_{r_i}^{r_o}(1-r_s/r)^{-1}dr = \frac{1}{c}\left(r_o - r_i + r_s \ln\frac{r_o - r_s}{r_i - r_s}\right) \qquad (1)$$

[8] (p. 130). Multiplication of this time interval by the standard velocity of light, c, then formally generates a distance:

$$c\Delta t = \left(r_o - r_i + r_s \ln\frac{r_o - r_s}{r_i - r_s}\right). \qquad (2)$$

This vertical distance near a black hole has no name so far. One sees that it diverges (becomes infinite) when r_i approaches the Schwarzschild radius r_s. This reflects the well-known fact implicit in Eq.(1) that light emerging from the Schwarzschild radius (at $r_i = r_s$) takes an infinite time to reach an outside point r_o and vice versa [8]. The reason is also well known: the speed of light, c as a function of r, approaches zero as r approaches r_s in the Schwarzschild metric [8].

What could be shown in reference [7] is that the distance described by Eq.(2) is *real*. That is, the infinite time it takes by Eq. (1) to cover the distance between r_s and r_o is in accord with the interpretation that c is *constant* throughout. This "constant-c interpretation" of Eq.(2) is compatible with a proposal made by Max Abraham in 1912 [9] in response to the first non-constant-c theory proposed by Einstein in 1911 [10]. The variable-c feature got then incorporated four years later into general relativity – which indeed might never have been found without it. Now, the variable-c property unexpectedly turns out to be redundant in a special case: Eq.(2) formally implies that closer and closer to the horizon, space gets more and more strongly dilated in compensation for the lacking decrease in c [7]. The same locally isotropic size change had been demonstrated before in the much more special context of the equivalence principle [11].

The new taking-literally of Eq.(2) is tantamount to an infinite downward-extension of the Einstein-Rosen funnel (the upper half of the famous Einstein-Rosen bridge). Three previously unknown facts follow from the re-interpretation of the unchanged mathematics: 1) infinite proper in-falling time; 2) infinitely delayed Hawking radiation; 3) infinitely weak chargedness of black holes. All 3 contradict accepted wisdom, so the standard calculations must have involved an undiscovered false step at some point since the mathematics is unchanged. Indeed for one of the three (the first), a straightforward proof could be found that the non-constant-c traditional interpretation makes the *same* prediction [7]. This proof, which invokes a standing vertical laser wave between r_o and r_s, can be extended to the other two.

So much for Eq.(2) and its implications. What are the consequences for miniblack holes? The validity of points (i–iv) above is affected. Firstly, point (i) – failure to be produced – becomes indistinuguishable from the other case that already formed miniholes remain *undetected*. Hence there is a certain risk now that the experiment will be unnecessarily cranked up to unwittingly produce heavier miniholes than intended.

Secondly, point (iv) gets altered by new matter-eating properties of miniblack holes: The *linear* growth rate asserted ceases to be valid since it was based on the assumption that the eating-up of a charged quark does not alter the chances of the *next* encounter. In contradistinction, the new unchargedness implies that a change in the environment is generated by every single eating act. Only when the minihole eats exactly one negative charge (electron) per positive charge (1.5 positive quarks) eaten is there no imbalance induced. But to assume such a symmetry would be unrealistic since the effective eating cross-sections differ for electrons and quarks due to their strongly differing speeds (classically speaking). To fix ideas, the encounter rates may be assumed to be much higher for the (on average) positively charged quarks. At first sight the pertinent implication of "one orphaned electron per day" sounds innocuous enough – a new eating rule cannot possibly modify the million-year prediction of point (iv), one feels. Nevertheless the *linear* prediction made under point (iv) above breaks down.[1)]

Nonlinearity comes in many forms. Some have no qualitative implications, others have a knack for "self-organization." This fact enabled the origin of life [12]. Could it be that nature harbors a second "pet attractor" – one that raises its head whenever a black hole with its nonlinear eating habits is placed into eatable matter? The answer to this question appears to be yes. Everyone has seen pictures of the most spectacular self-organizing nonlinearity in the cosmos: the beautiful saddle-focus – a disk of in-spiralling matter with an orthogonal giant jet of charged particles being ejected from the middle on either side – called a "quasar" [13]. *Quasars* typically contain a 1-billion-solar-mass black hole at the center of their in-spiralling accretion disk of matter and the jets of charged particles extend over millions of light years. But quasars do not stand alone: *Microquasars* look just the same even though they contain only a single star – after upscaling by a factor of one billion (cf. [13]). Can this astounding self-similar hierarchy – the only of its kind in nature – perhaps be continued downwards?

There is no doubt at present that a *planet* – roughly a million times smaller in mass again than a microquasar – will once more "look the same" downscaled by another factor of a million as a "picoquasar," if given the chance. Indeed at the end of the millions of years of linear growth predicted by point (iv) above, a violent continuation like this one was not exluded [3]. Once nonlinearity is acknowledged as putting an end to linear growth at *some* stage, however, the same onset can no longer be ruled out to occur at some *earlier* stage.

This is the "early self-organization hypothesis" of black hole growth. Although nothing but a possibility-in-principle, it compellingly implies that the down-scaling of quasars does not stop at the one-earth-mass level, or a tenth, or whatever. No one has any idea at present how far the *quasar-generating principle* continues down the line. What is certain is only that each further step implies a proportional "reduction of the linear waiting time" after which the minihole ceases to be a pussycat and becomes a planet-eating monster. The decrease in waiting time could make up a factor of a thousand or a factor of a million or very much more. No matter how trivial it may appear, this scaling-induced "acceleration" is our main result.

Let us be a bit more specific. Since we started out from a nonlinearity valid at the lowermost end of the ladder, the self-organizing attractor from the upper end can in principle be thought to downscale all the way through. The limit at the lower end would then be our minihole itself. A quasar replica this size, if existing, would be by three multiplicative factors of a billion smaller than a picoquasar is relative to a quasar. This maximally consistent self-organizational scenario is the *picocubed quasar conjecture*. How likely is it?

This is the ten-billion-dollar question. In order to assess the validity or not of this worst-case scenario, the active properties of black holes at both ends of the scale need to be known. On the one end, a good quasar theory is required, cf. [13], on the other, an equally detailed knowledge of the eating process performed by a near-Planck sized miniblack hole is called for. Even the first task is incompletely answered at present. Penrose for one believes in rotational energy to be transformed into radiation [14]. At the lower end of the scale, virtually nothing is known to date beyond the mere "existence of nonlinearity" as we saw. And the fact that quantum mechanics is bound to enter in a currently unknown way. Nothing but "educated guesses" are possible.

One (uneducated) guess goes like this: First, a point-shaped quark comes close enough to the minihole to get gravitationally captured if some third body (a confinement-partner perhaps) absorbs part of the joint angular momentum. An in-spiralling motion could then set in for the captured partner on a scale of 10^{-27} cm, perhaps. The circling charge would generate a strong magnetic field orthogonal to the disk. A nuclear electron could be magnetically attracted along the axis to stick close to the minihole during the time the quark circles down the upper part of the funnel after passing the Zel'dovich threshold. During this period, the minihole would be effectively negatively charged – attracting the next positively charged quark with a force 20 or 30 orders of magnitude stronger than its gravitational pull. The thereby captured second quark then attracts *another* nuclear electron along the same transversal axis of magnetic flux lines while the previous electron gets expelled by the new one along the same axis – a first "jet."

This fictitious scenario is but one of many possible "maximally overstretched down-scalings" of the still not completely deciphered self-similar "quasar-producing principle" of nature. It would be surprising if the self-organizational power of the combined electromagnetic and gravitational saddle focus called an "ideal quasar" indeed went that far. But to exclude even this extremal downward extension with certainty is apparently not possible at the time being. As mentioned, it does not really matter where precisely the process stops at the lower end. Although it would of course be as uncautious to insist on this "maximal nonlinear scenario" as to insist on the "maximal linear scenario" extracted from the first eating event. The picocubed quasar hypothesis, which reaches down almost to the level of a quantum-constrained electro-gravitational dynamo (EGD), is only the smallest and perhaps least likely member in a long series of most certainly valid brothers.

So much about the new situation valid in the face of nonevaporation and unchargedness implicit in Eq.(2). Is the emerging *unified quasar theory* perhaps linked to the soon-to-be-switched-on experiment at Geneva? All that can be said at the time being is that the over-optimistic millions-of-years *linear* scenario of point (iv) above is no longer on the table (cf. [2]). The alternative *nonlinear* theory offered (quasar-attractor hypothesis) is preliminary. Under ordinary circumstances, it would certainly not be made the basis of an experiment any time soon. But if the desire were there for some reason, the pertinent experiment would be even *more* extensive than the one soon to be launched – because as a first step a far-out space-platform would need to be built. This would at least double the cost compared to the earth-based endeavor.

No matter how defective, however, the above theory *is* currently about to be tested on earth: near the snow-mountain which according to the children's song harbors the spring of eternal youth. This brings us to the non-physical *third* part of the present analysis. The two new results presented above (Abraham scaling and quasar scaling) do go hand in hand with a third "scaling problem" if you so wish – of the time needed to evaluate the other two. This is because the new physical scaling implicit in general relativity's Eq.(2) comes from a group almost devoid of credentials in the field. Hence it could take years until the idea is given the benefit of the doubt. A second minus point is that the new idea seemingly leads to absurdities. The infinite distance of the horizon if true implies inaccessibility of the inner Schwarzschild solution including the famous singularity inside in defiance of 90 years of high-level work. It therefore goes without saying that the 6.000 scientists associated with CERN – almost the majority of the scientific community – will hardly see reason to wait until a learned debate about Eq.(2) has set in. The fact that the new theory is formally irrefutable since the mathematics is unchanged may or may not be sufficient reason to give it the benefit of the doubt. Even publishing may prove impossible in the short time window left. In such a case one can only hope that the theory in question is devoid of applications.

Perhaps it is. The new elephant-trunk feature of the Einstein-Rosen funnel could be hoped to prevent string theory from being applicable – so that the experiment loses all danger by not working. However, this desperate alternative between a beautiful theory on the one hand and a beautiful planet on the other is unlikely to be chosen by fate. For as is well known, Einstein returned time and again to string-like (Kaluza-type) theories in the 1920's and 30's. This compatibility between string theories and general relativity is most certainly inherited by the particular solution to the Einstein equations focused on here.

If the new interpretation of Eq.(2) is *both* correct *and* compatible with string theory, however, the planned experiment acquires the status of a risk – the gravest of history. This fact is unknown. Is there anything that can still be done to make the theory heard to the extent that it will be given the benefit of the doubt so we can all see that it is *false* (as I hope it to be)? There is one point left to mention even if this goes against the grain of scientific etiquette: the discreet charm of planet saving. A street ballad comes to mind as a conceptual bridge:

"Last time I tried to do something for the planet was thirteen years ago,
'Lampsacus hometown of all persons on the Internet' [15] was the name,
by coincidence the police were ordered-in
to keep me out of my lecture hall for months in a row,
three times was I carried out in front of the students,
each time the plain-clothes officer in charge spontaneously
apologized afterwards which invisibly restored dignity, but the
requisite 10 billion dollar fund never materialized."

Sorry if you are lost without the melody. But the last stanza contains the missing cue: Planet-saving involves a fund of 10 billion if the right of the young majority to get access to life-saving information and knowledge is to be met – in the one case. And Planet-saving frees a fund of 10 billion if CERN stops the experiment – in the other case. A superficial coincidence. Or is there a common denominator between the two street ballads invoked, the one with an ancient Greek name and the modern one at the lake? At any rate it goes without saying that CERN must *not* when idling turn over its fund to Lampsacus since it needs it to foot its other activities – not to mention the fact that CERN created the Internet in the first place and hence occupies a saintly status.

Nonetheless everyone sees that the planet suddenly is safe again. For everybody realizes as if personally taken by the shoulder that planet-saving *is* worth ten billion on the free market. So it goes without saying that at least one far-sighted sponsor will chip in the money on CERN's behalf – for Lampsacus. To celebrate the fact that humankind just escaped from grave danger. Thales of Miletus once demonstrated to his fellow citizens that there are situations in which theory is stronger than business. Business, in turn, is grateful for being given a planet. In this way a simple theory – a constant factor introduced into an old equation – can have world-saving repercussions (☺).

Even if false (☺☺).

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Footnote

[1)] The new unchargedness is not actually needed to arrive at this result, since a progressively more and more strongly charged miniblack hole would entail nonlinear implications, too.
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ref.[7]:

Abraham-like return to constant c in general relativity: "ℜ-theorem" demonstrated in Schwarzschild metric

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Abstract

General relativity allows for a mathematically equivalent version in which length changes absorb the traditional changes in c. This conjecture is deomonstrated for the special case of the radial Schwarzschild metric. Two size-change results obtained a decade ago in the context of the equivalence principle – one relativistic, one quantum – are re-obtained in the radial Schwarzschild metric. Hence a previously neglected radial observable defined by $d\Re/dr = 1/(1–2m/r)$ determines physical distance. Since $d\Re/dt \equiv c$, Max Abraham's constant-c postulate of 1912 is unexpectedly fulfilled. The well-known infinite "radar distance" of the horizon of a Schwarzschild black hole therefore reflects an infinite distance. An infinite proper infalling time into black holes is a corollary. Since the latter time is canonically finite, an *anomaly* is encountered. To help decide it, an independent second proof is sketched based on a standing vertical light wave. An added merely qualitative third proof involves the Finkelstein diagram. If the new result can be confirmed, finished black-hole horizons, wormholes, Hawking radiation, charged black holes and singularities cease to exist in nature. Quantum-supported linear and curvature-supported nonlinear features of spacetime can be distinguished. ElNaschie's fractal E-infinity theory offers itself as an independent test bed.
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1. Introduction

Einstein first introduced a height-dependent c (in a high tower on earth or equivalently an ignited long rocket in outer space) in the context of the equivalence principle in 1911 [1]. This proposal caused grave concern on the part of his elder colleague Abraham who, after having fully embraced Einstein's special relativity, was reluctant to sacrifice the latter's central tenet of a globally constant speed of light c [2]. Einstein's new axiom of a potential-dependent c was instrumental to further progress and got eventually incorporated into general relativity four years later, as is well known [3].

The variable-c axiom has a familiar consequence in the Schwarzschild metric, which is the single most important solution of the Einstein equation of 1915. The "coordinate speed of light" c(r) is here a function of the distance parameter r:

$$c(r) \;=\; \frac{dr}{dt} = c \cdot (1 - 2m/r) \;, \qquad (1)$$

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where 2m is the Schwarzschild radius (with $2m \equiv 2GM/c^2$, M the central mass, G Newton's gravitational constant and c the universal speed of light (cf. Foster and Nightingale [3], p. 129). Eq.(1) states that the speed of light valid with respect to the distance parameter r, c(r), becomes zero as r approaches the Schwarzschild radius 2m from above.

In spite of its well-known lack of constancy relative to r, c is bound to remain at least *locally* unchanged by virtue of Einstein's covariance postulate (which posits that locally, the laws of nature must be everywhere the same including the speed of light). That this constraint is indeed fulfilled by Eq.(1) can be seen as follows: Proper time τ is locally *slowed down* by the factor $(1–2m/r)^{1/2}$ relative to coordinate time t (since $d\tau = (1–2m/r)^{1/2}dt$; [3], p. 127). This is the same factor by which the radial distance R is locally *increased* relative to coordinate distance r (since $dR = (1–2m/r)^{–1/2}dr$; [3], p. 125). The two local changes – the temporal and the spatial one – taken together compensate for the change in c given by Eq.(1). Indeed $dR/d\tau = dR/dr \cdot dt/d\tau \cdot dr/dt = (1–2m/r)^{–1}\, dr/dt \equiv c$.

The *global* change in c formally implicit in Eq.(1) conflicts with Abraham's intuition. Could it be that, contrary to appearances, Abraham's postulate is actually *fulfilled* in the Schwarzschild metric, and if so in general relativity at large? The answer to this question is in the positive as far as the radial Schwarzschild metric is concerned. This surprise result is to be demonstrated in the following along with some implications.

2. The size-change conjecture

In 1998, an in principle well-known but rarely (if ever) mentioned relativistic fact was independently spotted in the equivalence principle: *inequality* of the two vertical radar distances (down-up and up-down, respectively) in an accelerating rocket [4]. The method used was the "WM-diagram." The two mirror-symmetric capital letters W and M stand for light rays moving updown or downup twice, respectively (forming a symmetric XXXX pattern). The diagram illustrates that *time intervals* along the top and the bottom of the 4 concatenated X's (that is, "upstairs" and "downstairs" in a vertically accelerating rocket) interlock consistently with each other *despite* their unequal durations. While this fact is well-known in principle (compare the "Einstein synchronization" of Rindler [5]), the pictorial method – which grew out of a chaos-theoretic mapping proposal made by Dieter Fröhlich – reveals a new fact: *relative size increase downstairs by the redshift factor observed from upstairs*. This is because the vertical distance, when measured using light pulses from upstairs, is exactly so much larger than when measured from downstairs. Conversely, the blueshift factor observed downstairs implies an equal *relative size decrease upstairs* by the blueshift factor observed downstairs, which amounts to the same thing. (The objection that *width* appears unchanged from the respective other vantage point can be met by invoking projective anisotropy.) The relative size change *explains* the unequal vertical radar distances found in the equivalence principle. The latter are, by the way, easy to verify empirically using a TV tower, a pocket laser, a mirror and a counter (Gerhard Schäfer, personal communication 2001). The *size change result* is, by the way, already implicit in a special-relativistic finding of Walter Greiner's [6].[1)]

In the same year 1998, Heinrich Kuypers came up with the idea to have a look, likewise in the equivalence principle, at the gravitational Dopplershift of *matter waves* in order to see how quantum mechanics fits in. This allowed him to realize that, if photon mass downstairs is reduced by the gravitational redshift factor as is well known [7], *any mass* on the same level must be reduced by the same factor owing to local energy conservation [8,9].[2)] Hence

quantum mechanics predicts (via the de Broglie wave-length of matter waves and, more specifically, the Bohr radius which is inversely proportional to electron mass) that the *size* of every object downstairs is enlarged in proportion to its redshift [8,9]. This quantum prediction *coincides* with the previous relativistic prediction in a kind of pre-established harmony.

The two 1998 observations were each made independently of Abraham's conjecture. *A priori* it appears infinitely unlikely to suspect a connection. Or could it be that Einstein and Abraham are *reconciled* by Fröhlich and Kuypers? It is this outlandish conjecture which is to be demonstrated in the following. Since the "playground" of the equivalence principle is no longer sufficient, the Schwarzschild metric offers itself as the ballpark of choice.

3. Demonstration of the conjecture

3.1 Some well-known findings

The Schwarzschild metric is the oldest explicit solution of the Einstein equation. It was already found in late 1915 by a friend of Einstein's under unfavorable personal circumstances (Karl Schwarzschild died soon thereafter). The mentioned book by Foster and Nightingale [3] will continue to be used in the following as a backdrop – with page numbers put in brackets, like (p. 130), referring to their book.

Just as it was the case before with the equivalence principle [4], the up-down and the down-up distances measured by light sounding ("radar distances") differ by the mutual redshift (or, in the opposite direction, blueshift) factor *also* in the radial Schwarzschild metric. This fact deserves to be looked at in more detail.

Firstly, the mutual redshift factor owes its existence to the unequal *proper times* valid upstairs and downstairs. "Proper time" τ is, as already mentioned in the Introduction, at every local r defined by

$$d\tau = (1-2m/r)^{1/2}\, dt \qquad (2)$$

if t is the coordinate time (p. 127).

Secondly, the "coordinate time difference" Δt between upstairs and downstairs depends on the *coordinate values* of the *o*uter (r_o) and *i*nner (r_i) radial position, on the one hand, and the local *coordinate speed of light c(r)* given by Eq.(1), on the other. Integration of Eq.(1) if written in the form $dt = c^{-1}(1-2m/r)^{-1}dr$, between r_i and r_o, yields the *coordinate time difference* valid for a down-up (or equivalently up-down) light signal:

$$\Delta t = \frac{1}{c}\int_{r_i}^{r_o} (1-2m/r)^{-1}\, dr \qquad (3)$$

(p. 129). Multiplication of this time interval by c formally generates a corresponding *distance*:

$$c\Delta t = \int_{r_i}^{r_o} (1-2m/r)^{-1}\, dr \ . \qquad (4)$$

This distance has *no name* up until now. (Only the indefinite version of the same integral is well-known under the name " *r** " in the Eddington-Finkelstein formalism [10], a fact that we shall come back to below.)

The distance given by Eq.(4) cannot be measured directly. It can only be evaluated on either end – where it is then automatically weighted by the local time-shrinking factor of Eq.(2). What comes out is the well-known "radar-sounding light distance" (as Foster and Nightingale call it [3], p. 130). The latter reads, when evaluated from the *upper* end r_o,

$$c\Delta t_o \;=\; d\tau_o/dt \cdot c\Delta t \;\;=\;\; (1–2m/r_o)^{1/2} \int_{r_i}^{r_o} (1-2m/r)^{-1}\, dr$$

or after integration

$$c\Delta t_o \;=\; (1–2m/r_o)^{1/2} \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \qquad (5)$$

(p. 130). One sees that this *down-up radar distance* – as it can be called – diverges (becomes infinite) as r_i approaches the Schwarzschild radius 2m from above.

In corresponding fashion, the opposite radar distance $c\Delta t_i$ valid at the *lower* end r_i is arrived at. It differs from the former only by the subscript (i instead of o) in the first bracket:

$$c\Delta t_i \;=\; (1–2m/r_i)^{1/2} \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \;. \qquad (6)$$

This *up-down radar distance* – as it can be called – unlike the former does *not* diverge when r_i (now the position of the observer) approaches the Schwarzschild radius 2m from above.

The *ratio* between the two different radar distances, Eq.(5) and Eq.(6), is

$$\frac{c\Delta t_o}{c\Delta t_i} \;=\; \left(\frac{1-2m/r_o}{1-2m/r_i} \right)^{1/2} \;. \qquad (7)$$

This ratio is the "WM result" of reference [4] valid in the Schwarzschild metric.

So much for some well-known facts in the radial Schwarzschild metric. Only the distinction made between "down-up" and "up-down" radar distance appears to be new.

3.2 Compatibility with the Fröhlich-Kuypers size change

The described facts of the Schwarzschild metric can now be *juxtaposed* with the surprise observation of Fröhlich and Kuypers – the redshift-proportional size-change principle – in order to see how well the latter fits in or whether it creates an incompatibility at some point which would then spell the end of the present approach.

The new point heuristically to absorb into the Schwarzschild metric is the redshift-proportional relative *size increase* downstairs predicted by Fröhlich and Kuypers in two

independent contexts. Does this feature if hypothetically introduced *contradict* the accepted facts in the Schwarzschild metric? Surprisingly, the answer is *no*.

To see this, it is first necessary to realize that the Schwarzschild metric already *contains* a height-dependent change in size (which by the way *likewise* fails to show up in the transverse direction owing to projective anisotropy when looked at from above or below). This *canonical radial size increase* reads, as already mentioned in the Introduction,

$$dR = (1-2m/r)^{-1/2}dr \qquad (8)$$

(p. 125). After integration, this generates the so-called "radial distance" between r_i and r_o:

$$\Delta R = \int_{r_i}^{r_o} (1-2m/r)^{-1/2}\, dr$$

(p. 128), or explicitly

$$\Delta R = \left[r_o(r_o-2m)\right]^{1/2} - \left[r_i(r_i-2m)\right]^{1/2} + 2m\ln\frac{r_o^{1/2}+\left(r_o^{1/2}-2m\right)^{1/2}}{r_i^{1/2}+\left(r_i^{1/2}-2m\right)^{1/2}} \quad . \qquad (9)$$

Note that this *traditional radial distance* does *not* diverge when r_i approaches the Schwarzschild radius 2m from above. Indeed, of the 4 radial distances identified so far in the Schwarzschild metric – Eqs.(4), (5), (6) and (9) –, only the first two diverge.

However, the "intrinsic local size change" dR, valid in the Schwarzschild metric with respect to the local distance parameter r by virtue of Eq.(8), is not the end of the story in our present context since there now possibly exists a *new* local size change – the one predicted by the above-mentioned combined WM and de-Broglie argument. This postulated new size change is governed by the relative redshift or blueshift valid at the respective other radial position. Hence it is determined by the ratio of frequency shifts, Eq.(7), *divided* by the local proper-time factor valid at the observing position r_o by virtue of Eq.(2). This yields the predicted net factor

$$\left(\frac{1-2m/r_o}{1-2m/r_i}\right)^{1/2} \cdot (1-2m/r_o)^{-1/2} \equiv (1-2m/r_i)^{-1/2}$$

for any object located at r_i observed from $r_o > r_i$. Thus, we have (writing r for r_i in the brackett)

$$d\rho = (1-2m/r)^{-1/2}\, dr \qquad (10)$$

as our conjectured new local size change factor.

The *postulated* new local size-change $d\rho$ of Eq.(10) has exactly the *same* form as the local size-change dR of Eq.(8) above. Therefore there are *two* possibilities open at this point: Either the new size change factor of Eq,(10) is nothing but a new re-derivation of the old factor of Eq.(8); then the traditional radial distance R of Eq.(9) remains the only physically relevant radial distance in the Schwarzschild metric. Or *both* size change factors (the old dR/dr and the new $d\rho/dr$) contribute on an equal footing locally if the new size change of Fröhlich and Kuypers is real. In this case the resulting "effective local size change factor" $d\Re/dr$ is equal the product of the two individual factors named:

$$\frac{d\Re}{dr} = \left| \frac{dR}{dr} \cdot \frac{d\rho}{dr} \right| = (1 - 2m/r)^{-1} \ ,$$

that is,

$$d\Re \ = \ (1–2m/r)^{-1} \, dr \ . \qquad (11)$$

This hypothetical new effective local size change factor generates a *new distance integral*:

$$\Delta\Re \ = \ \int_{r_i}^{r_o} (1 - 2m/r)^{-1} dr \ = \ \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \ . \qquad (12)$$

The new distance integral $\Delta\Re$ ("$\Re$-distance") *replaces* the traditional distance integral ΔR of Eq.(9) as the correct "radial distance" – *if* the new Fröhlich-Kuypers size change factor is added while everything else remains unchanged.

Unexpectedly, Eq.(12) is *identical* to Eq.(4) above. Thus *nothing* has been introduced in effect as far as measured distances are concerned! The above employed "roundabout way" of heuristically using *two* local size changes – the old Schwarzschild factor of Eq.(8) and the hypothetical new Fröhlich-Kuypers factor of Eq.(10) – in order to explain the old radar distance of Eq.(4) proves to be a perfectly legitimate option. This option renders the traditional position-dependent *reduction of c* of Eq.(1), which likewise leads to Eq.(4) (≡ Eq.12), redundant. Both views make equal sense at first sight. So one should let nature have a word. The new view if false should lead to predictions at variance with reality. Is this the case?

3.3 The Shapiro time delay

The Shapiro time delay was introduced in 1964 by Shapiro [11] and independently by Muhleman and Richley [12] as a testable counterintuitive implication of the Schwarzschild metric ("fourth test of general relativity"). They encountered much skepticism at first. To date, the underlying equation (Eq.3) is empirically confirmed in the solar system to an accuracy of $2 \cdot 10^{-5}$ [13]. The currently accepted interpretation is that time suffers a counterintuitive delay while the radial distance R is covered and that this delay is predictably caused by the slowing of the velocity of light c(r) near a gravitating object.

But there now exists an alternative interpretation: the new size change axiom of Eq.(11) can be invoked. Adopting this interpretation is equivalent to saying that it is "not a change in c but a change in distance" that has been measured. This means that the two identical distances, $c\Delta t$ of Eq.(4) and $\Delta\Re$ of Eq.(12), can both be re-named into a *single* distance,

$$R_A \ = \ \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \ . \qquad (13)$$

3.4 Abraham vindicated

The newly obtained unique distance R_A produces (after division by c) the very time delay Δt familiar from Eq.(3) above (with ensuing radar distances Eqs.5 and 6). The old local size

change factor of Eq.(8) valid in the Schwarzschild metric ceases to be alone since a new factor, Eq.(10), has been brought in.

That *both* factors are valid in the Schwarzschild metric (in the product of Eq.11) comes as a surprise from the point of view of the equivalence principle. Here it is not the new factor of Eq.(10) which is surprising but the fact that it no longer stands alone in determining size in the Schwarzschild metric due to the presence there of the *old* factor of Eq.(8). This amounts to a *qualitative difference* between the "curved" Schwarzschild metric and the "flat" equivalence principle. Quantum mechanics continues to "see" only the flat version and so do mass and energy. Only *size* (and with it distance) is determined by *both* factors.

If we accept the new size change law (Eq.11) as being valid in the Schwarzschild metric: what about Abraham's hunch? The new-old distance found (Eq.13) deserves to be given a new name: "Abraham distance" – R_A. Why? Because this distance (Eq.12≡Eq.4) formally implies that *c is constant* over the whole trip! This fact was already implicit in Eq.(4) above – but our eyes were held at the time as it were since we did not yet have a good reason to take the coordinate-time difference Δt of Eq.(3) *that* seriously.

The new "Abrahamian interpretation" of Eq.(13) is *equivalent* to the standard interpretation of the radial Schwarzschild metric – as far as predicted redshifts, time delays for light and any resulting formal distances are concerned – yet with *c globally constant*. Hence we can state the following "$\mathfrak{R}$ theorem":

Theorem: *In the radial Schwarzschild metric, global constancy of c holds true with respect to the* natural *distance parameter $\mathfrak{R}$, defined by* $d\mathfrak{R} = (1–2m/r)^{-1}dr$.

The *naturalness* follows from the Fröhlich-Kuypers size-change. The *validity* follows (using Eqs.11 and 1) from the identity $d\mathfrak{R}/dt = (1–2m/r)^{-1}dr/[dr(1–2m/r)^{-1}/c^{-1}] \equiv c$.[3)]

A more general way to put the same result would be to speak of the "conservation of longitudinal spacetime volume" (longitudinal spacetime area) in the radial Schwarzschild metric – and presumably general relativity at large. In the present context, the formulation that "Abraham's dream" is fulfilled for once in general relativity in the special case of the radial Schwarzschild metric, is perhaps the most appropriate.

4. Consequences

4.1 First, the familiar side

The unified picture arrived at does not change anything in the accepted facts. Only on the level of *interpretation* are there any consequences to expect. One such interpretational consequence is, nevertheless, quite tangible:

Corollary: *The horizon of a Schwarzschild black hole has an infinite distance R_A (≡$\mathfrak{R}$) from the outside.*

This infinite-distance result does not really come as a surprise because the "radar distance" (signal-return time multiplied by ½c) of the horizon is well-known to be infinite from above by virtue of Eq.(5) as we saw ([3], p. 130). In the present context, this familiar finding acquires a subtle change of meaning, however: The infinite radar distance is no longer an

"artifact" of the *change-in-c* downstairs, as had to be assumed up until now, but the consequence of a previously overlooked *change-in-size* downstairs. According to the achieved new semantics, the same distance thus is *really* infinite from above. This conclusion is in perfect agreement with the Abraham principle. Everything appears harmonized for once.

4.2 A surprise secondary implication

In spite of the harmony obtained, there exists a derived *secondary* implication which appears virtually unacceptable: Black holes can now no longer be reached in finite time – not only by *light* with its infinite radar-sounding delay for which this fact is well known as we saw (Eq.5 and Shapiro), but by *any* infalling object. The result is so strong it even remains true when the falling time is measured in terms of the *proper* time of the infalling object itself! For the relative distance is now "really infinite" (R_A is infinite for $r_i \equiv 2m$). Hence the above "change in semantics" is *more* than a mere change of words for once: it has tangible physical consequences. Since this cannot be the case by very definition, some previously accepted *physical facts* are bound to have been in error!

This statement amounts to an *anomalous situation* having been reached. Hence the anomalous "infinite proper infalling time" merits an independent proof in terms of the *standard picture* since the physics is bound to be invariant under a change of semantics. If such a proof were to be found, the accepted ways of deriving the contrary – dating back to Oppenheimer and Snyder's famous paper of 1939, cf. [14] – would lose credit. The at first sight more natural thing to do – to re-work the old equations themselves – would be counterproductive, given that the pertinent mathematical paths have all stood the test of time. Only a round-about way – like the cat's around the hot mush – has any chance to succeed in case there *really* is something out of kilter. Such an alternative proof can tentatively be based on the paradigm of a *standing light wave* (generated by two mutually opposite laser sources of perfectly matching frequency and phase, cf. [15]). A rough sketch goes as follows:

A standing light wave is assumed to be set up vertically between the horizon and the outside world. This can be achieved in principle: two mutually opposite laser canons of *differing* frequencies can be positioned upstairs and downstairs in such a way as to generate a standing light wave in between them – if the frequency ratio matches the mutual redshift or blueshift factor (Eq.7). (If necessary, mediating "doubly open laser canons" tuned to the locally matching intermediary frequency can be inserted.) In the extremal case – outside-to-horizon – at stake, the frequency ratio between downstairs and upstairs approaches infinity. In this limit, the resulting "Jacobian ladder of light" possesses an *infinite* number of rungs (standing wave-crests). This prediction is in accord with the accepted infinite radar distance (Eq.5). While the locally valid distance between rungs differs widely – approaching zero for people living near the horizon –, the distance between rungs is *constant* for a fictitious particle falling at constant speed. Note that according to the equivalence principle, a *light wave* sent down from above *retains* its frequency in the upper frame in spite of its being progressively shortened when arriving at – or passing by – a more downstairs position. The same features-conserving fact holds true for a constant-speed particle that is slower than a photon. However, the speed of a falling ordinary particle is *not* constant but accelerating by definition. The situation is exactly the same as it holds true for any other ladder of infinitely many equi-spaced rungs – in special relativity. In special relativity, an infinite number of equi-spaced wave crests *cannot* be passed by in finite proper time – neither at constant speed nor at constant acceleration nor (as here) under an increasing but flattening-out acceleration; compare Eq.(5.24) of French [16] with the pertinent classical exercise (20.2) of Greiner's

book ([6], p. 168). This result carries over via the equivalence principle. Hence the total proper infalling time is *infinite*. (Q.e.d.)

The result just sketched is in accord with the infinite distance of Eq.(13) above. Still, since the time-honored reigning consensus holds that the Schwarzschild metric implies a *finite* proper infalling time (cf. [3], p. 139, or [14], p. 851), a third, only *qualitative*, argument appears desirable to have as well:

Pictures come to mind at this point. More specifically, the fact that the "coordinate speed" of an infalling body, v(r) = dr/dt ([3], p. 143), needs to be constantly *adjusted* to the local "coordinate speed of light" c(r) = dr/dt of Eq.(1). This "consistency check" is particularly vital at coordinate values close to the horizon where the radial light cones become more and more compressed around the curves of infalling matter near the asymptotic vertical line r = 2m of the horizon, in the traditional r,t diagrams. While a detailed account of the local situation is not possible in such drawings, there is one exception: the Finkelstein diagram ([17], p. 152). Here, the ingoing light rays are straight 45-degree ascending lines that, nevertheless, are subject to a (graphically invisible) *exponential scaling* in the neighborhood of the vertical line r = 2m. The same applies to the almost parallel slightly less slanted particle rays. Since in this diagram, r*+t is plotted versus the horizontal r axis ([17], p. 150), the Finkelstein diagram is compatible with Eq.(13) above. For r* = R_A in this diagram (as mentioned above following Eq.4). Although this pictorial argument is only *qualitative*, as ordered, it can possibly even be made quantitative (q.e.d.).

4.3 Consequences of the new unreachability

Firstly: If the horizon cannot be *reached* in finite time by any object, black holes also can no longer even *form* in finite time. For a horizon that cannot be reached in finite time can also not arise in finite time. (What precisely happens when just the "last iota" of mass remains to be added to an almost-critical homogeneous collection of masses, represents an interesting selforganization-type question; note that action-at-a-distance cannot be invoked in this context.) From the nonexistence of a finished horizon it then follows that Hawking's beautiful evaporation result [18], which relies on a finished horizon, gets infinitely delayed, too, and hence ceases to be physically effective. This rule remains valid for mini-black holes (despite their greater tunneling capabilities) by virtue of Kuypers's quantum-scaling result.

Secondly: Light cones cease to be compressible in the radial direction of the Schwarzschild metric. This fact is bound to have further consequences – in the context of time machines and other very general implications of the Einstein equation (like gravitational waves and rotating frames). For example, wormhole-based time machines [19] depend on the horizon being reachable in finite time. They therefore are automatically ruled out in the Abraham picture. Gödel time machines, on the other hand, remain possible (compare the beautiful drawing in [17], p. 169). This fact notwithstanding, a cautioning remark recently offered by a youngster should perhaps not go unmentioned ("Time machines cannot exist given the infinite duration of the future." *Why?* "They would be all over the place by now." *Unless the percentage of time travellers that aren't infinitely careful about camouflage is zero.* "Yes – but this is unthinkable!").

4.4 Main open task

The revived Abraham proposal of a universal c amounts to a surprise implication of the radial Schwarzschild metric. Is it possible that alternative metrics derived from the Einstein

equation will teach otherwise? The mentioned qualitative fit with the Eddington-Finkelstein metric speaks in favor of reconciliation. Therefore, the next open task to solve reads: How do the field equations *themselves* look like if "size, not c" depends on the gravitational potential?

5. Discussion

A simple new result valid in a subcase of the Schwarzschild metric was presented. "Radial spacetime-volume conservation" is one possible way to put it. The slower the time locally, the larger space locally. The stronger the magnification of time, the stronger the magnification of space: hence "space-over-time" is constant – c. Max Abraham would have liked this result. A first glimpse of how his mind worked I got from Valérie and Christophe Letellier at the university of Rouen three years ago.

The result presented is nothing but a beginning. Nonradial directions in the Schwarzschild metric have yet to be considered. Angular momentum has to be introduced next (Kerr metric). And the full Einstein equation is waiting to be considered thereafter. Even more sophisticated higher-dimensional analogous equations [20,21] are bound to come next.

What will remain if the main result can be confirmed? Four results are likely to persist:
1) Nonexistence of finished horizons (due to an infinite proper infalling time) and hence nonexistence of finished black holes, so that only "almost black holes" [22] remain.
2) Nonexistence of Hawking radiation.
3) Nonexistence of any spacetime elements beyond the horizon (including singularities).
4) Nonexistence of charged almost black holes.

These four predictions are surprising because they each fly in he face of accepted wisdom. If they hold true in the radial Schwarzschild metric, analogous new results are bound to be found in the four less restricted cases mentioned. It hence would be nice to have a simple method to falsify the above result. An independent approach to quantum spacetime was found by ElNaschie [23], cf. [24]. It will be instructive to see whether part or all of the above predictions can be confirmed or disproved in this independent methodology.

To conclude, a so-called "variantological approach" to spacetime physics has been presented. That is to say, a fictitious return to an earlier level of sophistication was heuristically adopted [25]. Whether the presented approach can stand the test of time is open. Possibly – or hopefully –, it can be falsified soon since its results challenge too many accepted facts in the modern fabric of spacetime. A priori speaking, the probability that the two simple insights of Fröhlich and Kuypers can turn back the wheel of history to a time when Einstein and Abraham fought their friendly battle of giants is negligibly small. Where precisely is the error located?

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Footnotes

[1)]The size change can also be derived from the twin-clocks paradox of special relativity: Conservation of angular momentum implies that the "younger clock" (if implemented as a frictionless rotator) must have been proportionally *enlarged* while making its fewer turns. Cf. [8,9] for an analogous implication of the gravitational twin paradox.

[2)]The same fact was mentioned in passing by Werner Israel [26]: Quote: "the gravitational (..) redshift factor (..) recalibrates locally measured mass and work to energies available to an observer at infinity."

[3)]Note, by the way, the interesting identity $d\Re/dt \equiv dR/d\tau$ (see Introduction).

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